Linear Algebra and its Applications 432 (2010) 3337–3342

Contents lists available at ScienceDirect

Linear Algebra and its Applications

journal homepage: www.elsevier.com/locate/laa

Algebraic reflexivity of sets of bounded operators on vector valued Lipschitz functions ☆

Fernanda Botelho, James Jamison *

Department of Mathematical Sciences, The University of Memphis, Memphis, TN 38152, United States

ARTICLE INFO

Article history:
Received 21 August 2009
Accepted 14 January 2010
Available online 23 February 2010

Submitted by C.K. Li

AMS classification:
Primary: 47B33
Secondary: 47B37

Keywords:
Algebraic reflexivity
Generalized bi-circular projections

ABSTRACT

In this paper we establish algebraic reflexivity properties of subsets of bounded linear operators acting on spaces of vector valued Lipschitz functions. We also derive a representation for the generalized bi-circular projections on these spaces.

1. Introduction

Let (X, d) and (Y, D) be compact metric spaces, and $\varphi : X \to Y$. The function φ is Lipschitz if and only if there exists a positive constant K such that

$$(*) \quad D(\varphi(x_0), \varphi(x_1)) \leqslant Kd(x_0, x_1), \quad \text{for every } x_0 \text{ and } x_1 \text{ in } X.$$

The infimum of all numbers K for which the condition $(*)$ holds is called the Lipschitz constant of φ, and is denoted by $L(\varphi)$. Equivalently we have

$$L(\varphi) = \sup_{x_0 \neq x_1} \frac{D(\varphi(x_0), \varphi(x_1))}{d(x_0, x_1)}.$$

☆ The authors wish to thank the referee for a careful reading of the manuscript and for several helpful suggestions.

* Corresponding author.
E-mail addresses: mbotelho@memphis.edu (F. Botelho), jjamison@memphis.edu (J. Jamison).

0024-3795/$ - see front matter
doi:10.1016/j.laa.2010.01.026

A bijective function φ is a lipeomorphism if φ and φ^{-1} are Lipschitz functions. Similarly, we say that a function $f : X \to E$, from a compact metric space X to a Banach space $(E, \|\cdot\|_E)$ is Lipschitz if there exists $K > 0$ such that $\|f(x_0) - f(x_1)\|_E \leqslant Kd(x_0, x_1)$, for every $x_0, x_1 \in X$. We denote by $Lip(X, E)$ the Banach space of all E-valued Lipschitz functions on X with the norm $\|f\| = \max\{\|f\|_\infty, L(f)\}$, where $\|f\|_\infty = max_{x\in E}\|f(x)\|$ and $L(f) = sup_{x\neq y}\frac{\|f(x)-f(y)\|}{d(x,y)}$. Spaces of this type are well known and were first investigated by Johnson, see [8,9].

In [16], the authors give a characterization of linear isometries between Banach spaces of vector valued Lipschitz functions, see also [15,17]. This characterization is the main tool in our study of the algebraic reflexivity property for subsets of bounded linear operators on $Lip(X, E)$. Let f be a scalar valued Lipschitz function on X and $e \in E$, following the notation in [16], we define $f \otimes e : X \to E$ by $(f \otimes e)(x) = f(x)e$. Clearly $\|f \otimes e\| = \max\{\|f\|_\infty, L(f)\}\|e\|$. The space of all bounded linear operators on E is denoted by $\mathcal{B}(E)$ and S_E the unit sphere of E.

Theorem 1.1 (cf. [16]). *Let (X, d) and (Y, D) be compact metric spaces, E a strictly convex Banach space and e a unit vector in E. Let $T : Lip(X, E) \to Lip(Y, E)$ be a linear isometry such that $T(1_X \otimes e) = 1_Y \otimes e$.*

(1) *There exist a closed subset of Y, Y_0, a surjective Lipschitz function $\varphi : Y_0 \to X$ with*

$$L(\varphi) \leqslant \max\{1, \mathrm{diam}(X)/2\}$$

and a Lipschitz function $\tau : Y \to \mathcal{B}(E)$ such that $\tau(y) = T_y$, $\|T_y\| = 1$ for all $y \in Y$, and

$$T(f)(y) = T_y\left(f(\varphi(y))\right), \quad \textit{for all } f \in Lip(X, E) \textit{ and } y \in Y_0.$$

(2) *If T is surjective, then there exist a lipeomorphism φ from Y onto X with*

$$L(\varphi) \leqslant \max\{1, \mathrm{diam}(X)/2\} \textit{ and } L(\varphi^{-1}) \leqslant \max\{1, \mathrm{diam}(Y)/2\}$$

and a Lipschitz function $\tau : Y \to \mathcal{B}(E)$, such that $\tau(y) = T_y$ is a surjective isometry of E for all $y \in Y$, and

$$T(f)(y) = T_y\left(f(\varphi(y))\right), \quad \textit{for all } f \in Lip(X, E) \textit{ and } y \in Y.$$

Given a Banach space F, a subset $\mathcal{E} \subset \mathcal{B}(F)$ is algebraically reflexive if $T \in \mathcal{E}$ whenever $Tx \in \mathcal{E}x$, for every $x \in F$, cf. [11]. The Banach space F is said to be algebraically reflexive whenever the group of isometries on F, $\mathcal{G}(F)$, is algebraically reflexive. The authors have studied this problem for $F = C(X, E)$, with X a compact Hausdorff topological space and E a Banach space, see [2]. The algebraic reflexivity of various subsets and subspaces of $\mathcal{B}(F)$ has been the subject of study in several papers, see [3,7,10–14].

In this paper we consider the algebraic reflexivity problem for various subsets of $\mathcal{B}(Lip(X, E))$, with X a compact metric space and E a strictly convex Banach space. We show that certain subgroups of the isometry group are algebraically reflexive. We also investigate the algebraic reflexivity of subsets of generalized bi-circular projections. We recall that a projection P on a complex Banach space is a generalized bi-circular projection (gbp) if there exists a modulus 1 complex number λ ($\neq 1$) such that $T = P + \lambda(I - P)$ is an isometry, see [5], also [1,2]. Generalized bi-circular projections belong to the class of bi-contractive projections , i.e. those projections P such that $\|P\| = 1$ and $\|I - P\| = 1$, cf. [4]. In general, the characterization of bi-contractive projections on a given Banach space is a difficult and important problem, cf. [6]. Such characterization for the class of bi-contractive projections on $Lip(X, E)$ is an open question. In this paper, we obtain a representation for the gbps on $Lip(X, E)$. This representation is crucial in our study of the algebraic reflexivity problem for some natural sets of generalized bi-circular projections.

2. Algebraic reflexivity of the space of vector valued Lipschitz functions

We recall that X and Y are compact metric spaces and E is a strictly convex and complex Banach space. We denote by $\mathcal{G}(Lip(X, E), Lip(Y, E))$ the set of all isometries from $Lip(X, E)$ into $Lip(Y, E)$. Throughout this paper we fix e, some unit vector in E. The set $\mathcal{G}_e(Lip(X, E), Lip(Y, E))$ consists of all isometries T in $\mathcal{G}(Lip(X,E), Lip(Y,E))$ such that $T(1_X \otimes e) = 1_Y \otimes e$. Moreover, we denote by $\mathcal{SG}_e(Lip(X,E), Lip(Y,E))$ the subset of $\mathcal{G}_e(Lip(X,E), Lip(Y,E))$ consisting of all surjective isometries. If $X = Y$ we denote this set by $\mathcal{SG}_e(Lip(X,E))$.

In this section we show that if X is a connected n-dimensional manifold without boundary or X supports an injective real valued Lipschitz function, then $\mathcal{SG}_e(Lip(X,E))$ is algebraically reflexive under some constraints on E.

We first introduce the definition of e-locally surjective isometry.

Definition 2.1. Let $T : Lip(X,E) \to Lip(Y,E)$ be a bounded linear operator. T is an e-locally surjective isometry if and only if for every $f \in Lip(X,E)$ there exists $T^f : Lip(X,E) \to Lip(Y,E)$, a surjective isometry in $\mathcal{G}_e(Lip(X,E), Lip(Y,E))$, such that $T(f) = T^f(f)$.

The group $\mathcal{SG}_e(Lip(X,E))$ is said to be algebraically reflexive whenever every e-locally surjective isometry on $Lip(X,E)$ is surjective.

Theorem 2.1. *Let X be a compact metric space, Y a compact and connected n-manifold without boundary, E an algebraically reflexive and strictly convex Banach space. If $T : Lip(X,E) \to Lip(Y,E)$ is an e-locally surjective isometry, then T is a surjective isometry.*

Proof. Since T is an e-locally surjective isometry, for every $f \in Lip(X,E)$, there exists a surjective isometry $T^f \in \mathcal{G}_e(Lip(X,E), Lip(Y,E))$ such that $T(f) = T^f(f)$. In particular $T(1_X \otimes e) = T^{1_X\otimes e}(1_X \otimes e) = 1_Y \otimes e$, thus $T \in \mathcal{G}_e(Lip(X,E), Lip(Y,E))$.

Theorem 1.1 asserts the existence of a closed subspace Y_0, a surjective Lipschitz function $\varphi : Y_0 \to X$, a function $\tau : Y \to \mathcal{B}(E)$ such that $\tau(y) = T_y$, and

$$T(f)(y) = T_y(f(\varphi(y))), \quad \text{for all } y \text{ in } Y_0 \text{ and } f \in Lip(X,E).$$

For every $f \in Lip(X,E)$, Theorem 1.1 also implies the existence of a lipeomorphism φ_f and a family $\{T_y^f \mid y \in Y\}$ such that each T_y^f is a surjective isometry of E and :

$$T^f(g)(y) = T_y^f(g(\varphi_f(y))), \quad \text{for every } g \in Lip(X,E) \text{ and } y \in Y.$$

Hence, for $y \in Y_0$, we have

$$T_y(f(\varphi(y))) = T_y^f(f(\varphi_f(y))). \tag{1}$$

Let f be a constant function equal to ν, a nonzero vector in E. Eq. (1) becomes $T_y(\nu) = T_y^f(\nu)$. Hence T_y is a locally surjective isometry on E for every $y \in Y_0$. The algebraic reflexivity of E implies that, for $y \in Y_0$, T_y is a surjective isometry.

For a fixed $y_0 \in Y_0$, let ν be a nonzero vector in E and set $f(z) = d(z, \varphi(y_0))\nu$. Eq. (1) becomes

$$d(\varphi(y_0), \varphi(y_0))T_{y_0}(\nu) = d(\varphi_f(y_0), \varphi(y_0))T_{y_0}^f(\nu). \tag{2}$$

This implies that φ is injective. In fact, if there exist $y_1 \neq y_0$ such that $\varphi(y_0) = \varphi(y_1)$, Eq. (2) implies that $\varphi_f(y_0) = \varphi_f(y_1) = \varphi(y_0)$. This contradicts that φ_f is a lipeomorphism. Consequently, we have $\varphi : Y_0 \to X$ is a Lipschitz homeomorphism. Therefore $Y = Y_0$, since Y and Y_0 are homeomorphic and Y is a compact and connected n-manifold without boundary, see [2, p. 298]. It now remains to show that φ^{-1} satisfies a Lipschitz condition. If we assume otherwise, then there exist sequences $\{y_n\}$ and $\{z_n\}$ so that $y_n \neq z_n$ and

$$\lim_{n\to\infty} \frac{d(\varphi(y_n), \varphi(z_n))}{D(y_n, z_n)} = 0.$$

We set $\widetilde{Y} = \{(y,z) : y \neq z\}$ and $\beta\widetilde{Y}$ is the Stone-Čech compactification of $\widetilde{Y}$. Let $F : \widetilde{Y} \to \mathbf{R}$ be given by $F(y,z) = \frac{d(\varphi(y),\varphi(z))}{D(y,z)}$. We denote by βF the unique continuous extension of F to the Stone-Čech compactification of $\widetilde{Y}$, $\beta F : \beta\widetilde{Y} \to \mathbb{R}$. Since we are assuming that 0 is in the range of βF, let $\xi \in \beta\widetilde{Y} \setminus \widetilde{Y}$

such that $\beta F(\xi) = 0$. The compactness of $\beta\widetilde{Y}$, implies the existence of a net in $\widetilde{Y}$, $\{(y_\alpha, z_\alpha)\}_\alpha$ converging to ξ. To see this, we first notice that, without loss of generality, we may assume that $\{y_\alpha\}$ converges to y in Y and $z_\alpha \neq y$, for all α. We define the Lipschitz function $f(z) = d(\varphi(y), \varphi(z))\nu$, where ν is a nonzero vector in E. Eq. (1) reduces to

$$d(\varphi(z), \varphi(y))T_z(\nu) = d(\varphi_f(z), \varphi(y))T_z^f(\nu), \quad \text{for all } z \in Y.$$

Therefore

$$d(\varphi(z), \varphi(y))\|T_z(\nu)\| = d(\varphi_f(z), \varphi(y))\|T_z^f(\nu)\|.$$

We conclude that

$$d(\varphi(z), \varphi(y)) = d(\varphi_f(z), \varphi(y)) = d(\varphi_f(z), \varphi_f(y)). \tag{3}$$

Consequently, $\frac{d(\varphi(z_\alpha),\varphi(y))}{D(z_\alpha,y)} \geqslant [L(\varphi_f^{-1})]^{-1} > 0$, for all α. The uniform continuity of βF implies that $\beta F(\xi) \geqslant [L(\varphi_f^{-1})]^{-1}$. This contradiction shows that φ^{-1} is a Lipschitz function. □

Remark 2.1. The previous theorem also holds under the assumption that X and Y are compact metric spaces and there exists an injective real valued Lipschitz function g on X. To see this we note that without loss of generality we may assume that g is strictly positive. Given $y \in Y_0$, let u be a unit vector in E so that $\|T_y u\| = 1$. We set $f = g \otimes u$. Eq. (1) becomes

$$g(\varphi(y))T_y u = g(\varphi_f(y))T_y^f(u).$$

Therefore $g(\varphi(y)) = g(\varphi_f(y))$. The injectivity of g implies that $\varphi(y) = \varphi_f(y)$. The remainder of the proof follows as in the proof given for Theorem 2.1.

The next corollary is a straightforward consequence of Theorem 2.1 and Remark 2.1.

Corollary 2.1. *Let X be a compact metric space, E an algebraically reflexive and strictly convex Banach space.*

(1) *If there exists an injective real valued Lipschitz function on X, then $\mathcal{S}G_e(Lip(X,E))$ is algebraically reflexive.*
(2) *If X is an n-dimensional compact and connected manifold without boundary, then $\mathcal{S}G_e(Lip(X,E))$ is algebraically reflexive.*

We now study the algebraic reflexivity problem for certain subgroups of the group $\mathcal{S}G_e(Lip(X,E))$. We first prove a preliminary lemma. We consider an isometry T in $\mathcal{S}G_e(Lip(X,E))$, with representation $T(f)(x) = T_x(f(\varphi(x)))$, as described in Theorem 1.1.

We denote by G the group of all surjective isometries U on E such that $U(e) = e$. Let φ be a lipeomorphism on X. We set

$$\mathcal{S}G_G = \{S \in \mathcal{S}G_e(Lip(X,E)) :\ \exists U \in G \text{ and } i \in \mathbb{Z} \\ \text{s.t. } S(f)(x) = Uf(\varphi^i(x)),\ \forall f \in Lip(X,E) \text{ and } x \in X\}.$$

We observe that $\mathcal{S}G_G$ is a subgroup of $\mathcal{S}G_e(Lip(X,E))$.

Proposition 2.1. *Let E be an algebraically reflexive and strictly convex Banach space and G an algebraically reflexive subgroup of the group of all surjective isometries on E. If X is a compact metric space that supports an injective and real valued Lipschitz function then $\mathcal{S}G_G$ is algebraically reflexive.*

Proof. Let T be a bounded linear operator on $Lip(X,E)$ that is locally in $\mathcal{S}G_G$, i.e. for each $f \in Lip(X,E)$ there exists $S^f \in \mathcal{S}G_G$ such that $T(f) = S^f(f)$. Corollary 2.1 implies that $T \in \mathcal{S}G_e(Lip(X,E))$. Theorem 1.1 asserts that $T(f)(x) = T_x(f(\psi(x)))$. Therefore, for every f, there exists $U^f \in G$ and $j(f) \in \mathbb{Z}$ such that

$$T_x(f(\psi(x))) = U^f f(\varphi^{j(f)}(x)), \text{ for all } x \in X. \tag{4}$$

We denote by g an injective positive function defined on X and set $h = g \otimes e$. For each $x \in X$ we have $g(\psi(x)) = g(\varphi^{j(h)}(x))$ and $\psi(x) = \varphi^{j(h)}(x)$. Furthermore, given a constant function equal to $v \in E$, Eq. (4) reduces to $T_x v = U^v v$, for all $x \in X$. The statement now follows from the algebraic reflexivity of G. $\square$

3. Algebraic reflexivity of sets of generalized bi-circular projections

In this section we study algebraic reflexivity properties of classes of generalized bi-circular projections. We denote by $\mathcal{GBP}(F)$ the set of all generalized bi-circular projections on a Banach space F. We say that a projection P is a locally gbp if for every $u \in F$ there exists a gbp P_u such that $P(u) = P_u(u)$. Furthermore $\mathcal{GBP}(F)$ is said to be algebraically reflexive if every locally gbp is a gbp. We prove that $\mathcal{GBP}(Lip(X, E))$ is algebraically reflexive under some constraints on X and E. We also show that a subset of $\mathcal{GBP}$ consisting of those gbps that can be expressed as the average of the identity operator with an isometric reflection is algebraically reflexive. An isometry T is an isometric reflection if T^2 is the identity operator.

3.1. Generalized bi-circular projections on Hilbert spaces

We first establish the algebraic reflexivity of the set of all gbps on a Hilbert space. We start by proving that gbps on a Hilbert space are the hermitian projections.

Proposition 3.1. *Let E be a Hilbert space. P is a generalized bi-circular projection on E if and only if P is hermitian.*

Proof. Since P is a projection, we have that $E = Ran(P) \bigoplus Ker(P)$. We observe that $Ran(P) = \{v \in E : P(v) = v\}$. Moreover, there exists a modulus 1 complex number λ and a surjective isometry T such that $P + \lambda(I - P) = T$. This implies that T is equal to the identity when restricted to the range of P, and is equal to λI when restricted to the $Ker(P)$. If $u \in Ran(P)$ and $v \in Ker(P)$ then $T(u + v) = u + \lambda v$. Since T is an isometry we have that $Re((\lambda - 1)\langle v, u\rangle) = 0$ where $\langle \cdot, \cdot \rangle$ denotes the inner product in E. Then $(\lambda - 1)\langle v, u\rangle = ib$ for some $b \in \mathbb{R}$. It follows that $ibv \in Ker(P)$, hence $Re((\lambda - 1)\langle ibv, u\rangle) = -b^2 = 0$. This proves that P is orthogonal and thus hermitian. Conversely, if P is a hermitian projection then for every $u \in Ran(P)$ and $v \in Ker(P)$ we have $\langle u, v\rangle = 0$. Then, given an arbitrary vector $w \in E$, $w = u + v$ with $u \in Ran(P)$ and $v \in Ker(P)$. Let λ be a modulus 1 complex number ($\lambda \neq 1$), and define $T(w) = u + \lambda v$. The operator T is an isometry and $P = \frac{-\lambda I + T}{1 - \lambda}$. Hence P is a gbp. $\square$

Theorem 3.1. *If E is a complex Hilbert space then $\mathcal{GBP}(E)$ is algebraically reflexive.*

Proof. Since gbps on a Hilbert space are the hermitian projections, we show that hermitian projections are algebraically reflexive. We denote by $\langle \cdot, \cdot \rangle$ the inner product in E . Let P be a locally hermitian projection, ie. for every $u \in E$ there exists a hermitian projection Q_u such that $P(u) = Q_u(u)$. We show that P is hermitian by proving that $\langle u, P(u)\rangle$ is real for every vector $u \in E$. This is clear since $\langle u, P(u)\rangle = \langle u, Q_u(u)\rangle$ and Q_u is hermitian. $\square$

We also establish the algebraic reflexivity of certain projections defined on an algebraically reflexive Banach space. These projections are given as the average of the identity operator with an isometric reflection. We recall that an isometric reflection on a Banach space is an isometry with square equal to the identity. We observe that such projections are gbps associated with $\lambda = -1$.

Proposition 3.2. *If E is an algebraically reflexive Banach space the set of all projections on E given as the average of the identity with an isometric reflection is algebraically reflexive.*

Proof. Let P be a projection which is locally given by the average of the identity with an isometric reflection. For every $v \in E$ there exists an isometric reflection T_v such that $P(v) = \frac{v+T_v(v)}{2}$. Therefore $T = 2P - I$ is a locally surjective isometry. Since E is algebraically reflexive, T is a surjective isometry and P is the average of the identity with an isometric reflection. $\square$

3.2. Generalized bi-circular projections on $Lip(X, E)$

We now investigate the algebraic reflexivity properties of classes of gbps acting on $Lip(X, E)$. As previously stated, we assume that X is a compact metric space and E is a strictly convex Banach space.

We denote by $\mathcal{G}BP_e(Lip(X, E))$ the set of all gbps P on $Lip(X, E)$ such that $P(1_X \otimes e) = 1_X \otimes e$. The next theorem describes this class of generalized bi-circular projections on $Lip(X, E)$.

Theorem 3.2. *Let X be a compact metric space and E be a strictly convex Banach space. If P is a projection in $\mathcal{G}BP_e(Lip(X, E))$, then (1) $P = \frac{I+T}{2}$ with $T \in \mathcal{S}G_e(Lip(X, E))$ and $T^2 = I$, or (2) $P(f)(x) = P_x(f(x))$, with P_x a gbp on E and, for all $x \in X$, e in the range of P_x.*

Proof. Let P be a projection in $\mathcal{G}BP_e(Lip(X, E))$. Then there exist a unimodular complex number λ and a surjective isometry T such that $P + \lambda(I - P) = T$ and $T^2 - (\lambda + 1)T + \lambda I = 0$. Clearly $T \in \mathcal{S}G_e(Lip(X, E))$. Theorem 1.1 asserts the existence of a lipeomorphism $\varphi : X \to X$ and a Lipschitz function $\tau : X \to \mathcal{B}(E)$ given by $\tau(x) = T_x$, such that $T(f)(x) = T_x(f(\varphi(x)))$. If $\lambda = -1$ then $T_x T_{\varphi(x)} f(\varphi^2(x)) = f(x)$. This implies that $\varphi^2 = I$ and $T_x T_{\varphi(x)} = I$. Therefore $P = \frac{I+T}{2}$. This implies that $T = 2P - I$ and then $T^2 = I$, i.e. T is an isometric reflection. We assume that $\lambda \neq -1$. If $x_0 \neq \varphi(x_0)$, then there exists $f \in Lip(X, E)$ such that $f(\varphi(x_0)) = \nu$ (ν is a unit vector in E) and $f(x_0) = f(\varphi^2(x_0)) = 0$, see [18, Theorem 1.5.6 on p. 16]. This implies that $T_{x_0}(\nu) = 0$. This contradiction shows that $\varphi = I$ and, for every x, $P_x = \frac{-\lambda I+T_x}{1-\lambda}$ is a gbp on E. We also note that $P(f)(x) = P_x(f(x))$. $\square$

Remark 3.1. Proposition 3.2 also implies that the set of all gbps in $\mathcal{G}BP_e(Lip(X, E))$ given as the average of the identity with an isometric reflection on $Lip(X, E)$ is algebraically reflexive whenever $Lip(X, E)$ is algebraically reflexive.

References

[1] F. Botelho, J. Jamison, The algebraic reflexivity of $\ell^p(X)$, JMAA 346 (1) (2008) 141–144.
[2] F. Botelho, J. Jamison, The algebraically reflexivity of $C(X, E)$ and Cambern theorem, Studia Math. 186 (3) (2008) 295–302.
[3] S. Dutta, T.S.S.R.K. Rao, Algebraic reflexivity of some subsets of the isometry group, Linear Algebra Appl. 429 (2008) 1522–1527.
[4] P.K. Lin, Generalized bi-circular projections, JMAA 340 (1) (2008) 1–4.
[5] M. Fošner, D. Ilišević, C.K. Li, G-invariant norms and bi-circular projections, Linear Algebra Appl. 420 (2007) 596–608.
[6] Y. Friedman, B. Russo, Contractive projections on $C_0(K)$, Trans. AMS 273 (1) (1982) 57–73.
[7] K. Jarosz, T.S.S.R.K. Rao, Local isometries of function spaces, Math. Z. 243 (2003) 449–469.
[8] J. Johnson, Lipschitz spaces, Pacific J. Math. 51 (1974) 177–186.
[9] J. Johnson, Banach spaces of Lipschitz functions and vector valued Lipschitz functions, Trans. Amer. Math. Soc. 148 (1970) 147–169.
[10] L. Molnàr, B. Zalar, Reflexivity of the group of surjective isometries, Proc. Edinburgh Math. Soc. 42 (1999) 17–36.
[11] L. Molnàr, Selected Preserver Problems on Algebraic Structures of Linear Operators and on Function Spaces, Series: Lecture Notes in Mathematics 1895 (2007).
[12] C. Sànchez, The group of automorphisms of L_∞ is algebraically reflexive, Studia Math. 161 (2004) 19–32.
[13] C. Sànchez, Local isometries on spaces of continuous functions, Math. Z. 251 (4) (2005) 735–749.
[14] C. Sànchez, L. Molnàr, Reflexivity of the isometry group of some classical spaces, Rev. Mat. Iberoamericana 18 (2002) 409–430.
[15] A. Jiménez-Vargas, M. Villegas-Vallecillos, Into linear isometries between spaces of Lipschitz Functions, Houston J. Math. 34 (4) (2008) 1165–1184.
[16] A. Jiménez-Vargas, M. Villegas-Vallecillos, Linear isometries between spaces of vector-valued Lipschitz Functions, Proc. AMS 137 (4) (2009) 1381–1188.
[17] E. Mayer-Wolf, Isometries between Banach Spaces of Lipschitz Functions, Israel J. Math. 38 (1–2) (1981) 58–74.
[18] N. Weaver, Lipschitz Algebras, World Scientific Publishing Co., 1999.